

# A linear moose model with pairs of degenerate gauge boson triplets

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The possibility of a strongly interacting electroweak symmetry breaking sector, as opposed to the weakly interacting light Higgs of the Standard Model, is not yet ruled out by experiments. In this paper we make an extensive study of a deconstructed model (or “moose” model) providing an effective description of such a strong symmetry breaking sector, and show its compatibility with experimental data for a wide portion of the model parameter space. The model is a direct generalization of the previously proposed D-BESS model.

## I. INTRODUCTION

Among the problems still left open by the Standard Model (SM) of particle physics, the understanding of the exact mechanism that leads to the breaking of the electroweak symmetry at low energies is of particular importance. Besides the SM basic Higgs mechanism, still to be verified by experiments, possible alternative solutions to the problem are offered by extensions of the old technicolor (TC) theories, where the Higgs boson is realized as a composite state of strongly interacting fermions.

These theories have recently received a renewed attention starting from higher dimensional Lagrangians; effective chiral Lagrangians in four dimensions containing new resonance states can be obtained by the deconstruction technique [1, 2, 3, 4, 5, 6, 7, 8, 9] or as holographic versions of 5-dimensional (5D) theories in warped background [10, 11, 12].

Models have been proposed, working in the framework suggested by the AdS/CFT correspondence, which assume a  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge group in the 5D bulk, [13, 14, 15, 16, 17, 18, 19], or also simpler with a  $SU(2)$  in the bulk [20, 21, 22, 23, 24]. One of the main challenges of these models is the value of the  $S$  parameter [25, 26] or the related  $\epsilon_3 = g^2 S / (16\pi)$  [27, 28, 29]. Indeed, the experimental value of  $\epsilon_3$  is of the order of  $10^{-3}$  [12],

whereas the value naturally expected in TC theories is an order of magnitude bigger.

A delocalization of the fermionic fields into the bulk as in [17, 30], realized in the deconstructed version by allowing standard fermions to have direct couplings to all the moose gauge fields as in [31], leads to direct contributions to the electroweak parameters that can correct the bad behavior of the  $\epsilon_3$  parameter. The fine tuning which cancels out the oblique and direct contributions to  $\epsilon_3$  in each bulk point, that is from each internal moose gauge group, corresponds to the so called *ideal delocalisation* of fermions, [31, 32, 33, 34, 35]. Other solutions to get a suppressed contribution to  $\epsilon_3$  have been investigated, like the one suggested by holographic QCD, assuming that different five dimensional metrics are felt by the axial and vector states [36, 37, 38, 39]. However it has been shown recently that the backgrounds that allow a negative oblique contribution to  $\epsilon_3$  are pathological, since they require unphysical Higgs profile or higher dimensional operators [40].

An alternative solution to the  $\epsilon_3$  problem was proposed in [41, 42, 43] (see also [44]). The solution was realized in terms of an effective TC theory of non linear  $\sigma$ -model scalars and massive gauge fields. The model is a four site model with three sigma fields. The physical spectrum consists of three massless scalar fields (the Goldstone bosons giving masses to the gauge vector particles) and two triplets of massive vector fields degenerate in mass and couplings. This model, named degenerate BESS (Breaking Electroweak Symmetry Strongly) model (D-BESS), has an enhanced custodial symmetry such as to allow  $\epsilon_3 = 0$  at the lowest order in the electroweak interactions. This idea has been also recently used in phenomenological analysis of low scale technicolor theories with vector and axial vector resonances very close in mass [45].

A generalization of D-BESS was studied in [22]. This extended model is a moose model, with a replicated  $SU(2)$  gauge symmetry, that maintains the most useful feature of D-BESS, namely the custodial symmetry which guarantees the vanishing of  $\epsilon_3$ .

Two other quantities,  $\epsilon_1$  and  $\epsilon_2$  [27, 28, 29] or equivalently  $T$  and  $U$  [25, 26], are customarily used to parameterize the electroweak precision observables. More recently, an alternative parametrization was proposed in terms of seven parameters [12] which describe in a very general way the effects of so-called “universal” extensions of the SM, that is theories whose deviations from the SM are all contained in the vector boson self-energies.

In the present paper, we wish to extend the calculation made in [22] by deriving the seven parameters of ref. [12], and from them the  $\epsilon$  parameters, to the next-to-leading order

in  $M_W^2/M^2$ , where  $M$  is the mass scale of the new bosons, and without any expansion in their gauge couplings. Also, we will calculate, at the same order, the trilinear gauge boson vertex anomalous contributions due to the new physics.

In Section II we review the notations and the main constitutive elements of the model. In Section III we derive the low energy effective Lagrangian by eliminating the fields of the internal moose and show that the model decouples in the limit  $M \rightarrow \infty$ . In Section IV we compute the effective gauge boson correlation functions, and from them the parameters of ref. [12] and the  $\epsilon$  parameters, to the next-to-leading order in  $M_W^2/M^2$ ; we then derive bounds on the model parameter space from experimental data. In Section V we obtain the effective trilinear gauge couplings to order  $M_W^2/M^2$ . Finally, in Section VI we present our conclusions.

## II. A LINEAR MOOSE MODEL FOR THE ELECTROWEAK SYMMETRY BREAKING

Our model is based on the idea of dimensional deconstruction [1, 2, 3, 4] and on the hidden gauge symmetry approach, historically applied both to strong interactions [8, 46, 47, 48] and to electroweak symmetry breaking [41, 42, 43, 49, 50].

Consider  $K+1$  non linear  $\sigma$ -model scalar fields  $\Sigma_i$ ,  $i = 1, \dots, K+1$  and  $K$  gauge groups,  $G_i$ ,  $i = 1, \dots, K$  with global symmetry  $G_L \otimes G_R$ . Since we are interested in studying the electroweak symmetry breaking mechanism, we will assume  $G_i \equiv SU(2)$ ,  $G_L \otimes G_R = SU(2)_L \otimes SU(2)_R$ . The transformation properties of the fields are

$$\begin{aligned}\Sigma_1 &\rightarrow L\Sigma_1 U_1^\dagger, \\ \Sigma_i &\rightarrow U_{i-1}\Sigma_i U_i^\dagger, \quad i = 2, \dots, K, \\ \Sigma_{K+1} &\rightarrow U_K \Sigma_{K+1} R^\dagger,\end{aligned}\tag{1}$$

with  $U_i \in G_i$ ,  $i = 1, \dots, K$ ;  $L \in G_L$ ,  $R \in G_R$ ; the Lagrangian is given by

$$\mathcal{L} = \sum_{i=1}^{K+1} f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2} \sum_{i=1}^K \text{Tr}[(\mathbf{F}_{\mu\nu}^i)^2],\tag{2}$$

where  $f_i$  are  $K+1$  free constants (“link” coupling constants). The covariant derivatives are

defined as follows:

$$\begin{aligned}
D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 + i \Sigma_1 g_1 \mathbf{A}_\mu^1, \\
D_\mu \Sigma_i &= \partial_\mu \Sigma_i - i g_{i-1} \mathbf{A}_\mu^{i-1} \Sigma_i + i \Sigma_i g_i \mathbf{A}_\mu^i, \quad i = 2, \dots, K, \\
D_\mu \Sigma_{K+1} &= \partial_\mu \Sigma_{K+1} - i g_K \mathbf{A}_\mu^K \Sigma_{K+1},
\end{aligned} \tag{3}$$

as implied by the transformation rules (1), where  $\mathbf{A}_\mu^i$  and  $g_i$  are the gauge fields and gauge coupling constants associated to the groups  $G_i$ ,  $i = 1, \dots, K$ .  $\mathbf{F}_{\mu\nu}^i$  has the standard definition

$$\mathbf{F}_{\mu\nu}^i = \partial_\mu \mathbf{A}_\nu^i - \partial_\nu \mathbf{A}_\mu^i + i g_i [\mathbf{A}_\mu^i, \mathbf{A}_\nu^i], \tag{4}$$

with

$$\mathbf{A}_\mu^i = A_\mu^{i,a} \frac{\tau^a}{2}. \tag{5}$$

Notice that one could introduce an additional field,

$$U = \Sigma_1 \Sigma_2 \cdots \Sigma_{K+1} \tag{6}$$

which transforms just like the usual chiral field of the Higgsless SM:  $U \rightarrow L U R^\dagger$ . The field  $U$  is an invariant under the  $G_i$  transformations (which are then effectively "hidden" to  $U$ ).

As shown in [22], in this model, due to the presence of a custodial  $SU(2)$  symmetry, we get no corrections to  $\epsilon_{1,2}$ . Also, if we put one (and only one) of the link coupling constants  $f_i$  equal to zero, we effectively enlarge the global symmetry to  $(SU(2) \otimes SU(2))^{K+1}$ , getting  $\epsilon_3 = 0$  too. We want to explore this particular case; also, for simplicity, we impose an extra left-right symmetry of the moose which identifies the two ends:

$$f_i \equiv f_{K+2-i}, \tag{7}$$

$$g_i \equiv g_{K+1-i}. \tag{8}$$

The reflection symmetry, together with the condition that just one of the link coupling constants must vanish, implies that the number of moose links has to be odd (and hence the number of gauge fields has to be even), and that the vanishing link has to be the central one. So we will consider:

$$K = 2N, \tag{9}$$

$$f_{N+1} = 0. \tag{10}$$

It is instructive to count out the number of degrees of freedom. Before cutting the central link, we had  $(2N + 1)$  matrices of scalar fields, for  $3(2N + 1)$  degrees of freedom, and  $3(2N)$  massless vector fields, for  $6(2N)$  degrees of freedom; of these, only 3 scalar fields are physical, the others disappearing (in the unitary gauge) to give mass to all the gauge bosons via the Higgs mechanism. After the cutting, we only get  $3(2N)$  scalar degrees of freedom to start with, so that no one survives the Higgsing of the gauge bosons.

It will be useful for further considerations to look at the form of the gauge boson mass matrix, which can be obtained by putting  $\Sigma_i = I$  in eq. (2). We find

$$\mathcal{L}_{\text{mass}} = \sum_{i=1, i \neq N+1}^{2N+1} f_i^2 \text{Tr}[(g_{i-1} \mathbf{A}_\mu^{i-1} - g_i \mathbf{A}_\mu^i)^2] \equiv \sum_{i,j=1}^{2N} (M_2)_{ij} \text{Tr}[\mathbf{A}_\mu^i \mathbf{A}^{j\mu}], \quad (11)$$

with

$$(M_2)_{ij} = g_i^2(f_i^2 + f_{i+1}^2)\delta_{i,j} - g_i g_{i+1} f_{i+1}^2 \delta_{i,j-1} - g_j g_{j+1} f_{j+1}^2 \delta_{i,j+1}, \quad (12)$$

$$i, j = 1, \dots, 2N, \quad g_0 = g_{2N+1} = 0.$$

Thanks to the condition  $f_{N+1} = 0$  and to the reflection symmetry, the matrix  $M_2$  is block diagonal with two degenerate blocks. Each block can be independently diagonalized through an orthogonal transformation  $S$ . By calling  $\tilde{\mathbf{A}}_\mu^i$ ,  $i = 1, \dots, N$  the mass eigenstates, and  $m_n^2$  the squared mass eigenvalues, we have

$$\mathbf{A}_\mu^i = \sum_{n=1}^N S_n^i \tilde{\mathbf{A}}_\mu^n, \quad (13)$$

with

$$S_m^i (M_2)_{ij} S_n^j = m_n^2 \delta_{m,n}, \quad (14)$$

and an analogous result holds for  $\mathbf{A}^i$ ,  $i = N + 1, \dots, 2N$ . We will assume  $m_n \neq 0$ ,  $n = 1, \dots, 2N$ , otherwise the model describes an unphysical situation.

### III. CALCULATION OF THE EFFECTIVE LAGRANGIAN

We will now switch on the electroweak interactions by gauging the  $SU(2)_L \otimes U(1)_Y$  subgroup of the global  $G_L \otimes G_R$ . We will include in the model only standard fermions coupled to  $SU(2)_L \otimes U(1)_Y$ . Then, considering the limit of heavy mass for the extra gauge bosons, we will integrate them out in order to obtain an effective description in terms of the electroweak and the fermion fields only.

Note that just promoting part of the global symmetry to a gauge symmetry is not enough to describe realistic  $W$  and  $Z$  bosons: we also need to provide suitable mass terms for three out of four of the newly added gauge fields. In the model as it is, however, there is not any scalar degree of freedom left to trigger a Higgs mechanism, as we have seen. A natural way out is to add to the Lagrangian an additional term containing the chiral field  $U$ , which obeys the transformation rule  $U \rightarrow LUR^\dagger$ :

$$\mathcal{L}_U = f_0^2 \text{Tr}[D_\mu U^\dagger D^\mu U], \quad (15)$$

with

$$\begin{aligned} D_\mu U &= \partial_\mu U - i\tilde{g}\tilde{\mathbf{W}}_\mu U + iU\tilde{g}'\tilde{\mathbf{Y}}_\mu, \\ \tilde{\mathbf{W}}_\mu &= \tilde{W}_\mu^a \frac{\tau^a}{2} \quad \tilde{\mathbf{Y}}_\mu = \tilde{Y}_\mu \frac{\tau^3}{2}. \end{aligned} \quad (16)$$

The  $U$  field gives us the additional three degrees of freedom we need and provides a SM-like symmetry breaking term for the gauge bosons  $\tilde{W}$  and  $\tilde{Y}$  associated to  $SU(2)_L \otimes U(1)_Y$ .

Summing up, the Lagrangian of the bosonic sector of the generalized D-BESS (GD-BESS) model is

$$\begin{aligned} \mathcal{L} &= \sum_{i=1, i \neq N+1}^{2N+1} f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] + f_0^2 \text{Tr}[D_\mu U^\dagger D^\mu U] \\ &\quad - \frac{1}{2} \text{Tr}[(\mathbf{F}_{\mu\nu}^{\tilde{W}})^2] - \frac{1}{2} \text{Tr}[(\mathbf{F}_{\mu\nu}^{\tilde{Y}})^2] - \frac{1}{2} \sum_{i=1}^{2N} \text{Tr}[(\mathbf{F}_{\mu\nu}^i)^2] \end{aligned} \quad (17)$$

where:

$$\begin{aligned} \mathbf{F}_{\mu\nu}^{\tilde{W}} &= \partial_\mu \tilde{\mathbf{W}}_\nu - \partial_\nu \tilde{\mathbf{W}}_\mu + i\tilde{g}[\tilde{\mathbf{W}}_\mu, \tilde{\mathbf{W}}_\nu], \\ \mathbf{F}_{\mu\nu}^{\tilde{Y}} &= \partial_\mu \tilde{\mathbf{Y}}_\nu - \partial_\nu \tilde{\mathbf{Y}}_\mu \end{aligned} \quad (18)$$

and the covariant derivatives of  $\Sigma_1$  and  $\Sigma_{2N+1}$  are modified as follows:

$$\begin{aligned} D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 - i\tilde{g}\tilde{\mathbf{W}}_\mu \Sigma_1 + i\Sigma_1 g_1 \mathbf{A}_\mu^1, \\ D_\mu \Sigma_{2N+1} &= \partial_\mu \Sigma_{2N+1} - ig_{2N} \mathbf{A}_\mu^{2N} \Sigma_{2N+1} + i\tilde{g}' \Sigma_{2N+1} \tilde{\mathbf{Y}}_\mu \end{aligned} \quad (19)$$

due to the gauging of  $SU(2)_L \otimes U(1)_Y$ .

The fermion interactions will be given by SM-like terms:

$$\begin{aligned} \mathcal{L}_{fermion} &= -\tilde{g} \bar{\psi} \gamma^\mu \frac{(1-\gamma^5)}{2} \frac{\tau^a}{2} \psi \tilde{W}_\mu^a - \tilde{g}' \bar{\psi} \gamma^\mu \frac{(1-\gamma^5)}{2} \frac{B-L}{2} \psi \tilde{Y}_\mu \\ &\quad - \tilde{g}' \bar{\psi} \gamma^\mu \frac{(1+\gamma^5)}{2} \left( \frac{B-L}{2} + \frac{\tau^3}{2} \right) \psi \tilde{Y}_\mu \end{aligned} \quad (20)$$

where  $\psi$  is a generic fermion doublet, and  $B, L$  are the barion and lepton numbers respectively. In this way, the new gauge bosons are coupled to the fermions only through their mixing with the SM ones.

By expanding eq. (17) in the unitary gauge  $\Sigma_i = I, \forall i$ , and separating the kinetic term contribution from that of the terms containing the link coupling constants, we get

$$\mathcal{L}_{kin} = -\frac{1}{4} \sum_{i=0}^{2N+1} (A_{\mu\nu}^{i,a} - g_i \epsilon^{abc} A_{\mu}^{i,b} A_{\nu}^{i,c})^2, \quad (21)$$

$$\mathcal{L}_{link} = \sum_{\substack{i=1 \\ (i \neq N+1)}}^{2N+1} \frac{f_i^2}{2} (g_{i-1} A_{\mu}^{i-1,a} - g_i A_{\mu}^{i,a})^2 + \frac{f_0^2}{2} (\tilde{g} \tilde{W}_{\mu}^a - \tilde{g}' \tilde{Y}_{\mu}^a)^2, \quad (22)$$

where we have made the identifications:

$$A_{\mu}^{0,a} = \tilde{W}_{\mu}^a, \quad A_{\mu}^{2N+1,3} = \tilde{Y}_{\mu}, \quad A_{\mu}^{2N+1,1} = A_{\mu}^{2N+1,2} = 0, \quad g_0 = \tilde{g}, \quad g_{2N+1} = \tilde{g}' \quad (23)$$

and defined:

$$A_{\mu\nu}^{i,a} = \partial_{\mu} A_{\nu}^{i,a} - \partial_{\nu} A_{\mu}^{i,a}, \quad i = 0, \dots, 2N+1. \quad (24)$$

The model field content is summarized in Fig. 1. For  $N = 1$  the model reduces to the D-BESS model [42, 43].

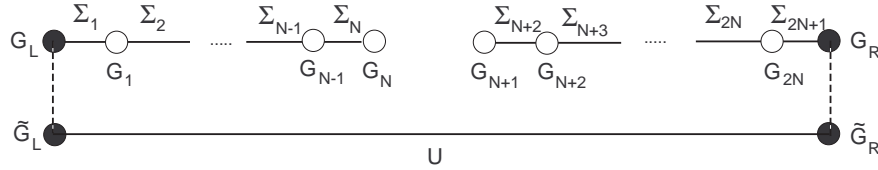


FIG. 1: Moose diagram for GD-BESS; it consists of a linear moose with the central link cut and a nonlocal connection (the  $U$  field) between the two end points of the moose.  $\tilde{G}_L \otimes \tilde{G}_R$  is the global symmetry group after the gauging of the electroweak subgroup.

From the Lagrangian (17), we can derive the classical equations of motion for the  $\mathbf{A}_{\mu}^i$  fields:

$$\begin{aligned} \partial_{\mu} \mathbf{F}^{i\mu\nu} &= ig_i [\mathbf{A}_{\mu}^i, \mathbf{F}^{i\nu\mu}] + g_i [f_i^2 (g_{i-1} \mathbf{A}^{i-1\nu} - g_i \mathbf{A}^{i\nu}) \\ &\quad - f_{i+1}^2 (g_i \mathbf{A}^{i\nu} - g_{i+1} \mathbf{A}^{i+1\nu})], \quad i = 1, \dots, 2N \end{aligned} \quad (25)$$

where again we have identified

$$\mathbf{A}_{\mu}^0 = \tilde{\mathbf{W}}_{\mu}, \quad \mathbf{A}_{\mu}^{2N+1} = \tilde{\mathbf{Y}}_{\mu}. \quad (26)$$

If this model is to be consistent with the existing experimental data, the masses of the  $A^{i,a}$  fields must be significantly larger than those of the SM gauge bosons. Let's call  $M$  the common mass scale of the heavy gauge bosons. Here we will be concerned only about the low energy predictions of the model, in which the new particles are not directly produced, but rather manifest themselves only by small modifications of the SM gauge boson propagators. So we will use an effective Lagrangian approach and work in the limit  $p^2 \ll M^2$ , where  $p$  represents the typical momentum scale of the processes we wish our effective theory to describe. Since the mass spectrum cannot be determined analytically in the general case, we do not have an exact expression for  $M$  but we can give an estimate of it by looking at the mass matrix (12). We see that all the terms in (12) are a sum of contributions which are proportional to  $f_i^2 g_j^2$  for some  $i, j$ ; so we can assume that the typical mass scale will be of order  $f_i g_j$  (we will get a more explicit estimate for the mass scale in the following). This assumption can actually be checked in the simplest case  $N = 1$ , as shown by the direct analysis made in ref. [42]. In this case, we have four massive eigenstates; their masses are, to lowest order:  $\tilde{M}_W \simeq \tilde{g} f_0$ ,  $\tilde{M}_Z \simeq \tilde{g}' f_0$  for the two lightest eigenstates, which can be identified with the SM gauge bosons, and  $M \simeq g_1 f_1$  for the two heaviest states, which are degenerate.

As a consequence, the limit we will study is

$$p^2 \ll f_i^2 g_j^2, \quad i = 1, \dots, N, N+2, \dots, 2N+1; \quad j = 1, \dots, 2N. \quad (27)$$

If we now rewrite eq. (25) as

$$\partial_\mu g_i \mathbf{F}^{i\mu\nu} + i[g_i \mathbf{A}_\mu^i, g_i \mathbf{F}^{i\mu\nu}] = g_i^2 [f_i^2 (g_{i-1} \mathbf{A}^{i-1\nu} - g_i \mathbf{A}^{i\nu}) - f_{i+1}^2 (g_i \mathbf{A}^{i\nu} - g_{i+1} \mathbf{A}^{i+1\nu})], \quad (28)$$

we can see that the quantities on the left-hand side are of higher order with respect to those on the right-hand side. Keeping only leading order terms, the equations reduce to

$$f_i^2 (g_{i-1} \mathbf{A}_\mu^{i-1} - g_i \mathbf{A}_\mu^i) - f_{i+1}^2 (g_i \mathbf{A}_\mu^i - g_{i+1} \mathbf{A}_\mu^{i+1}) = 0, \quad (29)$$

which imply

$$g_N \mathbf{A}_\mu^N = g_{N-1} \mathbf{A}_\mu^{N-1} = \dots = g_1 \mathbf{A}_\mu^1 = \tilde{g} \tilde{\mathbf{W}}_\mu \quad (30)$$

and

$$g_{N+1} \mathbf{A}_\mu^{N+1} = g_{N+2} \mathbf{A}_\mu^{N+2} = \dots = g_{2N} \mathbf{A}_\mu^{2N} = \tilde{g}' \tilde{\mathbf{Y}}_\mu. \quad (31)$$



Substituting this leading order expressions in the unitary gauge Lagrangian given in eqs. (21),(22), limiting us for the moment to the bilinear terms, we get:

$$\begin{aligned}\mathcal{L}_{eff}^2 = & -\frac{1}{2}\left(\frac{1}{\tilde{g}^2} + \frac{1}{\tilde{G}^2}\right)\tilde{g}^2\tilde{W}_{\mu\nu}^+\tilde{W}^{-\mu\nu} - \frac{1}{4}\left(\frac{1}{\tilde{g}^2} + \frac{1}{\tilde{G}^2}\right)\tilde{g}^2\tilde{W}_{\mu\nu}^3\tilde{W}^{3\mu\nu} - \frac{1}{4}\left(\frac{1}{\tilde{g}'^2} + \frac{1}{\tilde{G}^2}\right)\tilde{g}'^2\tilde{Y}_{\mu\nu}\tilde{Y}^{\mu\nu} \\ & + \tilde{g}^2 f_0^2 \tilde{W}_\mu^+ \tilde{W}^{-\mu} + \frac{\tilde{g}^2 f_0^2}{2} \tilde{W}_\mu^3 \tilde{W}^{3\mu} + \frac{\tilde{g}'^2 f_0^2}{2} \tilde{Y}_\mu \tilde{Y}^\mu - f_0^2 \tilde{g} \tilde{g}' \tilde{W}_\mu^3 \tilde{Y}^\mu,\end{aligned}\quad (32)$$

where

$$\frac{1}{\tilde{G}^2} = \sum_{k=1}^N \frac{1}{\tilde{g}_k^2} = \sum_{k=N+1}^{2N} \frac{1}{\tilde{g}_k^2} \quad (33)$$

and we have introduced the charged gauge fields  $\tilde{W}^\pm = \frac{1}{\sqrt{2}}(\tilde{W}^1 \mp i\tilde{W}^2)$ . This expression exactly reproduces the SM electroweak gauge Lagrangian, provided we rescale the fields  $\tilde{W}$ ,  $\tilde{Y}$ :  $\tilde{g}\tilde{W} \rightarrow g\tilde{W}$ ,  $\tilde{g}'\tilde{Y} \rightarrow g'\tilde{Y}$ , and identify

$$\frac{1}{g^2} = \left(\frac{1}{\tilde{g}^2} + \frac{1}{\tilde{G}^2}\right), \quad \frac{1}{g'^2} = \left(\frac{1}{\tilde{g}'^2} + \frac{1}{\tilde{G}^2}\right), \quad f_0^2 = \frac{v^2}{4} \equiv \frac{(\sqrt{2}G_F)^{-1}}{4}, \quad (34)$$

with

$$f_0^2 g^2 \simeq M_W^2, \quad f_0^2 (g^2 + g'^2) \simeq M_Z^2 \quad (35)$$

in the limit  $M \rightarrow \infty$ . This means that any deviation from the SM at low energy will be suppressed at least by a factor  $\frac{p^2}{M^2}$ .

We can now get the next-to-leading order expression for the  $\mathbf{A}^i$  iteratively, by substituting the leading order solutions (30)-(31) in the left-hand side of eq. (28). We get

$$g_i \mathbf{A}_\nu^i = g \tilde{\mathbf{W}}_\nu - c_i \mathbf{K}_\nu, \quad i = 1, \dots, N; \quad (36)$$

$$g_i \mathbf{A}_\nu^i = g' \tilde{\mathbf{Y}}_\nu - c_i \mathbf{H}_\nu, \quad i = N+1, \dots, 2N; \quad (37)$$

where we have introduced:

$$\begin{aligned}\mathbf{K}_\nu &= g \partial^\mu \mathbf{F}_{\mu\nu}^{\tilde{W}} + i g^2 [\tilde{\mathbf{W}}^\mu, \mathbf{F}_{\mu\nu}^{\tilde{W}}], \quad \mathbf{H}_\nu = g' \partial^\mu \mathbf{F}_{\mu\nu}^{\tilde{Y}}, \\ c_i &= \sum_{j=1}^i \frac{1}{f_j^2} \sum_{k=j}^N \frac{1}{g_k^2} = c_{N+i} = \sum_{j=N+i}^{2N+1} \frac{1}{f_j^2} \sum_{k=N+1}^j \frac{1}{g_k^2}, \quad i = 1, \dots, N.\end{aligned}\quad (38)$$

Notice that the  $c_i$  are positive definite and of order  $O(\frac{1}{M^2})$ , and the reflection symmetry implies  $c_i = c_{2N+1-i}$ .

Let us make the substitution for the  $\mathbf{A}^i$ . Limiting us to the quadratic part of the Lagrangian and using eqs. (36)-(37), we get:

$$\begin{aligned} \mathcal{L}_{eff}^2 = & -\frac{1}{2}\tilde{W}_{\mu\nu}^+\tilde{W}^{-\mu\nu} - \frac{1}{4}\tilde{W}_{\mu\nu}^3\tilde{W}^{3\mu\nu} - \frac{1}{4}\tilde{Y}_{\mu\nu}\tilde{Y}^{\mu\nu} \\ & + \frac{v^2g^2}{4}\tilde{W}_\mu^+\tilde{W}^{-\mu} + \frac{v^2g^2}{8}\tilde{W}_\mu^3\tilde{W}^{3\mu} + \frac{v^2g'^2}{8}\tilde{Y}_\mu\tilde{Y}^\mu - \frac{v^2gg'}{4}\tilde{W}_\mu^3\tilde{Y}^\mu \\ & + \frac{1}{4\overline{G}^2}\frac{1}{\overline{M}^2}\left(2g^2\tilde{W}_{\mu\nu}^+\square\tilde{W}^{-\mu\nu} + g^2\tilde{W}_{\mu\nu}^3\square\tilde{W}^{3\mu\nu} + g'^2\tilde{Y}_{\mu\nu}\square\tilde{Y}^{\mu\nu}\right); \end{aligned} \quad (39)$$

where

$$\frac{1}{\overline{M}^2} = C\overline{G}^2, \quad C \equiv \sum_{i=1}^N \frac{c_i}{g_i^2} \equiv \sum_{i=N+1}^{2N} \frac{c_i}{g_i^2}. \quad (40)$$

$\overline{M}$  can be used as an explicit estimate for the scale  $M$  (from the definition of the  $c_i$  in (38), we see that  $\overline{M}$  is indeed of order  $f_i g_j$ ).

#### IV. EFFECTIVE GAUGE BOSON CORRELATION FUNCTIONS AND $\epsilon$ PARAMETERS

From eq. (39), it is straightforward to calculate the correlators for the fields  $\tilde{W}$  and  $\tilde{Y}$ . Up to the fourth power of the momentum, we get

$$\begin{aligned} \Pi_{+-}(p^2) &= -\frac{1}{g^2} \left( g^2 \frac{v^2}{4} - p^2 - p^4 \frac{g^2}{\overline{M}^2 \overline{G}^2} \right) \\ \Pi_{33}(p^2) &= -\frac{1}{g^2} \left( g^2 \frac{v^2}{4} - p^2 - p^4 \frac{g^2}{\overline{M}^2 \overline{G}^2} \right) \\ \Pi_{YY}(p^2) &= -\frac{1}{g'^2} \left( g'^2 \frac{v^2}{4} - p^2 - p^4 \frac{g'^2}{\overline{M}^2 \overline{G}^2} \right) \\ \Pi_{3Y}(p^2) &= \frac{v^2}{4}. \end{aligned} \quad (41)$$

It is immediate to verify that, as it should,

$$\frac{1}{g^2} = \Pi'_{+-}(0), \quad \frac{1}{g'^2} = \Pi'_{YY}(0), \quad v^2 = -4\Pi_{+-}(0), \quad (42)$$

where the derivatives of the  $\Pi$  are taken with respect to  $p^2$ .

Following ref. [12], one can consider seven form factors, encoding the corrections of new physics to the electroweak precision observables:

$$\begin{aligned} \hat{S} &= g^2 \Pi'_{3Y}(0), \quad \hat{T} = g^2 M_W^2 (\Pi_{33}(0) - \Pi_{+-}(0)) \\ \hat{U} &= -g^2 (\Pi'_{33}(0) - \Pi'_{+-}(0)), \quad V = \frac{1}{2} g^2 M_W^2 (\Pi''_{33}(0) - \Pi''_{+-}(0)) \\ X &= \frac{1}{2} g g' M_W^2 \Pi''_{3Y}(0), \quad Y = \frac{1}{2} g'^2 M_W^2 \Pi''_{YY}(0), \quad W = g^2 M_W^2 \Pi''_{33}(0). \end{aligned} \quad (43)$$

Notice that the analysis of ref. [12] only applies to “universal” theories; the GD-BESS model belongs to this class since the couplings with the fermions are of the standard form. In our model, from the equality of  $\Pi_{+-}$  and  $\Pi_{33}$  and the expression for  $\Pi_{3Y}$  in eq. (41) it follows that  $\hat{S} = \hat{T} = \hat{U} = V = X = 0$ ; so we have only two non vanishing form factors, namely  $W$  and  $Y$ :

$$W = \frac{g^2 M_W^2}{\overline{M}^2 \overline{G}^2}, \quad Y = \frac{g'^2 M_W^2}{\overline{M}^2 \overline{G}^2}. \quad (44)$$

We can now compare the GD-BESS model predictions to experimental results. For this purpose, it is convenient to consider the  $\epsilon$  parameters, since they are better constrained by the data and more widely used in the literature. From the definition of the  $\epsilon$  in terms of the form factors [12], we get, as contributions from new physics,

$$\begin{aligned} \epsilon_1 &= \hat{T} - W + 2 \frac{s_\theta}{c_\theta} X - \frac{s_\theta^2}{c_\theta^2} Y, \\ \epsilon_2 &= \hat{U} - W + 2 \frac{s_\theta}{c_\theta} X - V, \\ \epsilon_3 &= \hat{S} - W + \frac{1}{s_\theta c_\theta} X - Y, \end{aligned} \quad (45)$$

where  $\tan(\theta) = g'/g$ . For the GD-BESS model we find:

$$\epsilon_1 = -\frac{(c_\theta^4 + s_\theta^4)}{c_\theta^2} \overline{X}, \quad \epsilon_2 = -c_\theta^2 \overline{X}, \quad \epsilon_3 = -\overline{X}, \quad (46)$$

with  $\overline{X}$  given by

$$\overline{X} = \frac{M_Z^2}{\overline{M}^2} \left( \frac{g}{\overline{G}} \right)^2. \quad (47)$$

As we can see, after the gauging of the electroweak interactions, the new physics contribution to the  $\epsilon$  parameters is no longer equal to zero, but the leading non vanishing order of the correction is  $O(M_Z^2/\overline{M}^2)$ . This contribution can be understood as follows: the weak interactions explicitly break the custodial  $(SU(2) \otimes SU(2))^{2N+1}$ , which protects  $\epsilon_3 = 0$ , down to the standard  $SU(2)_L \otimes U(1)_Y$ . The case with no weak interactions can be re-obtained in the limit  $M_Z^2/\overline{M}^2 \rightarrow 0$ , which represents a zeroth-order approximation of the model. More generally, all of the SM electroweak sector will be modified by  $O(M_Z^2/\overline{M}^2)$  contributions due to the new physics. As an explicit example of this, in the following Section V we will calculate the effective contribution to the trilinear gauge boson couplings.

The new physics contribution to the  $\epsilon$  parameters can be tested against experimental data. In Figure 2 we show a  $\chi^2$  contour plot in the plane  $(\overline{M}, \frac{1}{\overline{G}})$  at 95% C.L.; the contour

is obtained by considering the following experimental values for the  $\epsilon$  parameters:

$$\begin{aligned} \epsilon_1 &= (+5.0 \pm 1.1)10^{-3} \\ \epsilon_2 &= (-8.8 \pm 1.2)10^{-3} \\ \epsilon_3 &= (+4.8 \pm 1.0)10^{-3} \end{aligned} \quad \text{with correlation matrix} \quad \begin{pmatrix} 1 & 0.66 & 0.88 \\ 0.66 & 1 & 0.46 \\ 0.88 & 0.46 & 1 \end{pmatrix}; \quad (48)$$

and adding to the present model contributions the SM values:

$$\epsilon_1 = 3.4 \cdot 10^{-3}, \quad \epsilon_2 = -6.5 \cdot 10^{-3}, \quad \epsilon_3 = 6.7 \cdot 10^{-3}, \quad (49)$$

given for  $m_t = 170.9$  GeV and assuming an effective  $m_H = 1000$  GeV (experimental data are taken from [12] for the  $\epsilon$  parameters and from the Tevatron EWWG web site for the top mass, while the SM radiative corrections are obtained as a linear interpolation from the values listed in [51]). Notice that a relatively low scale  $\overline{M}$  is still allowed by present data. This is due to the double suppression factor present in  $\overline{X}$ , eq. (47). Notice also that the GD-BESS model in the limit  $\overline{M} \rightarrow \infty$  reproduces the SM to all orders in  $\frac{g}{G}$  (decoupling).

The result can be made much more explicit in the simplest case  $f_i \equiv f_c$  e  $g_i \equiv g_c \forall i$ . Recalling that

$$\frac{1}{\overline{M}^2} = C\overline{G}^2 = \sum_{i=1}^N \frac{c_i}{g_i^2} \left( \sum_{j=1}^N \frac{1}{g_j^2} \right)^{-1}$$

we have, in this case:

$$\overline{X} = \frac{N(N+1)(2N+1)}{6f_c^2 g_c^2} \left( \frac{g}{g_c} \right)^2 M_Z^2; \quad (50)$$

and, from  $\overline{X}$ , we immediately derive the  $\epsilon$  parameters from eq. (46). We can verify that with the substitutions  $f_c^2 \rightarrow 2a_2 \frac{v^2}{4}$  and  $g_c^2 \rightarrow \frac{g''^2}{2}$ , with  $N = 1$ , eqs. (46), (47) coincide with the D-BESS result [42, 43]. Indeed, in this case we find that

$$\overline{X} = \frac{1}{2a_2 \frac{v^2}{4} \frac{g''^2}{2}} \left( \frac{g}{g''} \right)^2 M_Z^2 = 2 \frac{M_Z^2}{M_{BESS}^2} \left( \frac{g}{g''} \right)^2, \quad (51)$$

where  $M_{BESS} = \sqrt{a_2} \frac{v}{2} g''$  coincides with  $\overline{M}$  for  $N = 1$  and, as shown in [42, 43], represents the mass of the two degenerate new resonances. Therefore for  $N = 1$  the limit shown in Fig. 2 can be interpreted as a bound on the degenerate masses of the new gauge vectors.

In general in order to get limitations from the electroweak precision data on the mass spectrum, one needs to perform the mass diagonalization which depends on the specific value of  $N$  and also on the particular choices of  $g_i$  and  $f_i$ . However, for any  $N$ ,  $\overline{M}$  gives the

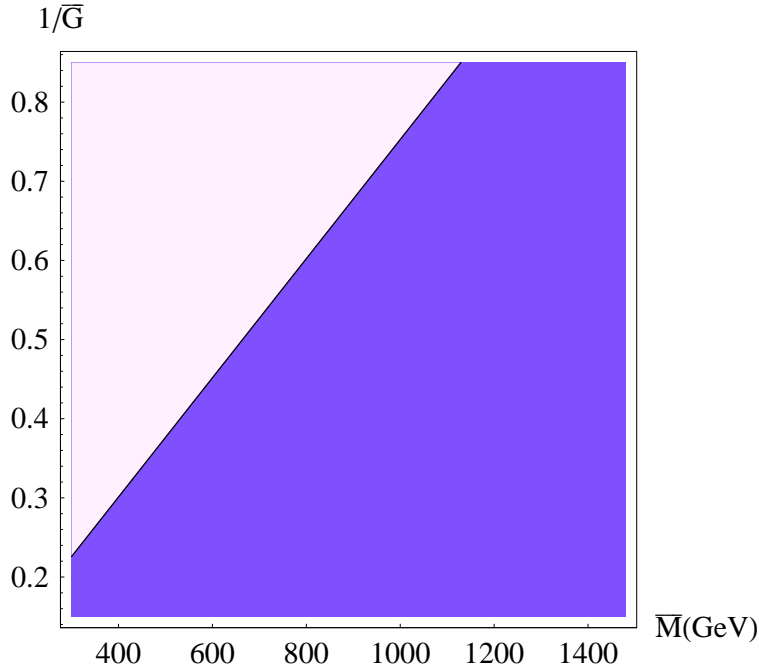


FIG. 2: 95% C.L. allowed region (the darker one), in the parameter space  $(\overline{M}, 1/\overline{G})$  by comparison of GD-BESS model predictions to electroweak precision parameters  $\epsilon_1$ ,  $\epsilon_2$  and  $\epsilon_3$ . Predictions include radiative corrections as in the SM with  $m_t = 170.9$  GeV and  $m_H = 1000$  GeV.

typical mass scale of the lowest resonance. For example, for  $g_i = g_c$ ,  $f_i = f_c$ , using the result for the spectrum given in [20], neglecting the electroweak interactions, the relation between the lightest charged resonance mass and  $\overline{M}$  is

$$M^{(1)} = 2 \sin\left(\frac{\pi}{2(N+1)}\right) \sqrt{\frac{(N+1)(2N+1)}{6}} \overline{M} \sim 1.6 - 1.8 \overline{M} \quad \text{for } N \gg 1 \quad (52)$$

Let us conclude this Section with a comment on the partial wave unitarity violation. A special feature of GD-BESS model is that the unitarity bound is completely determined by the  $U$  term given in eq. (15). In fact one can verify explicitly that the scattering amplitudes for the longitudinal electroweak vector bosons (using the equivalence theorem) are equal to the ones obtained for the Higgsless SM (for  $N = 1$  see [52]) and that all the amplitudes for the longitudinal  $A_i$ 's can be always arranged to have a higher unitarity bound [53]. Therefore this model is expected to become strongly interacting around a scale  $4\sqrt{\pi}v \sim 1.7$  TeV. One possible way to unitarize the GD-BESS model is to include also scalars associated to the  $\Sigma_i$  fields on each site and to the  $U$  field. In the simple case of  $N = 1$  we have shown

that the resulting theory is renormalizable (and unitary) and decoupling holds [54]. The generalization to generic  $N$  is under study.

## V. TRILINEAR COUPLINGS TO THE NEXT-TO-LEADING ORDER

We will now calculate, still to the next-to-leading order in the weak interactions, the contributions of GD-BESS model to the SM trilinear couplings. These can be read out of the effective trilinear Lagrangian, which is again obtained by substituting eqs. (36) and (37) in eqs. (21), (22). We get

$$\begin{aligned} \mathcal{L}_{eff}^3 = & -ig \left\{ \left[ \tilde{W}_{\mu\nu}^3 \tilde{W}^{+\mu} \tilde{W}^{-\nu} + \tilde{W}_\mu^3 (\tilde{W}^{-\mu\nu} \tilde{W}_\nu^+ - h.c.) \right] \right. \\ & - \frac{g^2}{G^2 M^2} \left[ (\square_3 + \square_+ + \square_-) (\tilde{W}_{\mu\nu}^3 \tilde{W}^{+\mu} \tilde{W}^{-\nu} + \tilde{W}_\mu^3 (\tilde{W}^{-\mu\nu} \tilde{W}_\nu^+ - h.c.)) \right. \\ & - \partial_\mu \tilde{W}_\nu^3 (\partial^\mu \partial_\rho \tilde{W}^{+\rho} \tilde{W}^{-\nu} - h.c.) + \partial_\mu \partial_\rho \tilde{W}^3{}^\rho (\partial^\mu \tilde{W}_\nu^+ \tilde{W}^{-\nu} - h.c.) \\ & - \tilde{W}_\mu^3 (\partial_\nu \tilde{W}^{+\mu} \partial^\nu \partial_\rho \tilde{W}^{-\rho} - h.c.) + \partial_\mu \tilde{W}_\nu^3 (\partial^\nu \partial_\rho \tilde{W}^{+\rho} \tilde{W}^{-\mu} - h.c.) \\ & \left. \left. - \partial_\mu \partial_\rho \tilde{W}^3{}^\rho (\partial_\nu \tilde{W}^{+\mu} \tilde{W}^{-\nu} - h.c.) + \tilde{W}_\mu^3 (\partial^\mu \tilde{W}_\nu^+ \partial^\nu \partial_\rho \tilde{W}^{-\rho} - h.c.) \right] \right\}, \end{aligned} \quad (53)$$

where  $\square_+$ ,  $\square_-$ ,  $\square_3$  operate only on the fields which bear the same index.

In order to make explicit the new physics anomalous contributions to the trilinear couplings, we will rewrite eq. (53) in terms of the mass eigenstates. First of all, we introduce  $\tilde{A}$  and  $\tilde{Z}$  fields from  $\tilde{W}^3$  and  $\tilde{Y}$  in the usual way:

$$\begin{pmatrix} \tilde{W}^3 \\ \tilde{Y} \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \tilde{Z} \\ \tilde{A} \end{pmatrix}. \quad (54)$$

Substituting in eq. (39) we get

$$\begin{aligned} \mathcal{L}_{eff}^2 = & -\frac{1}{2} \tilde{W}_{\mu\nu}^+ \tilde{W}^{-\mu\nu} - \frac{1}{4} \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} - \frac{1}{4} \tilde{A}_{\mu\nu} \tilde{A}^{\mu\nu} \\ & + \tilde{M}_W^2 \tilde{W}_\mu^+ \tilde{W}^{-\mu} + \frac{\tilde{M}_Z^2}{2} \tilde{Z}_\mu \tilde{Z}^\mu + \frac{1}{2\tilde{M}^2} [z_w \tilde{W}_{\mu\nu}^+ \square \tilde{W}^{-\mu\nu} \\ & + \frac{z_\gamma}{2} \tilde{A}_{\mu\nu} \square \tilde{A}^{\mu\nu} + \frac{z_z}{2} \tilde{Z}_{\mu\nu} \square \tilde{Z}^{\mu\nu} + z_{z\gamma} \tilde{A}_{\mu\nu} \square \tilde{Z}^{\mu\nu}] \end{aligned} \quad (55)$$

where

$$\begin{aligned} z_w = \frac{g^2}{G^2}, \quad z_\gamma = \frac{2e^2}{G^2}, \quad \text{with } e = gs_\theta = g'c_\theta, \\ z_z = \frac{g^2(c_\theta^4 + s_\theta^4)}{c_\theta^2 G^2}, \quad z_{z\gamma} = \frac{gg'c_{2\theta}}{G^2}, \quad \tilde{M}_W^2 = \frac{v^2 g^2}{4}, \quad \tilde{M}_Z^2 = \frac{v^2 (g^2 + g'^2)}{4}. \end{aligned} \quad (56)$$

We then introduce the field rescaling:

$$\begin{aligned}\tilde{W}_\mu^\pm &= \left(1 + \frac{z_w}{2} \left(\frac{\square}{M^2} - \frac{\tilde{M}_W^2}{M^2}\right)\right) W_\mu^\pm, \\ \tilde{Z}_\mu &= \left(1 + \frac{z_z}{2} \left(\frac{\square}{M^2} - \frac{\tilde{M}_Z^2}{M^2}\right)\right) Z_\mu, \\ \tilde{A}_\mu &= \left(1 + \frac{z_\gamma}{2} \frac{\square}{M^2}\right) A_\mu + z_{z\gamma} \frac{\square}{M^2} Z_\mu,\end{aligned}\tag{57}$$

which allows us to get rid of the anomalous “ $\square$ ” terms in the quadratic part of the Lagrangian. We then obtain

$$\mathcal{L}_{eff}^2 = -\frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} + M_W^2 W_\mu^+ W^{-\mu} + \frac{M_Z^2}{2} Z_\mu Z^\mu,\tag{58}$$

where:

$$M_W^2 = \tilde{M}_W^2 \left(1 - z_w \frac{\tilde{M}_W^2}{M^2}\right), \quad M_Z^2 = \tilde{M}_Z^2 \left(1 - z_z \frac{\tilde{M}_Z^2}{M^2}\right).\tag{59}$$

This is just the SM electroweak gauge boson bilinear Lagrangian; however, the rescaling (57) of the fields will affect both the couplings with fermions and the trilinear bosonic couplings. Let’s shift to the  $\tilde{W}^\pm$ ,  $\tilde{A}$ ,  $\tilde{Z}$  basis in eq. (20), then rescale the fields according to (57). We get:

$$\begin{aligned}\mathcal{L}_{charged} &= -\frac{e}{\sqrt{2}s_\theta} \bar{\psi}_u \gamma^\mu \left(1 - \frac{\gamma^5}{2}\right) \left(1 + \frac{z_w}{2} \left(\frac{\square}{M^2} - \frac{\tilde{M}_W^2}{M^2}\right)\right) \psi_d W_\mu^+ + h.c. \\ \mathcal{L}_{neutral} &= -\frac{e}{s_\theta c_\theta} \left(1 + \frac{z_z}{2} \left(\frac{\square_Z}{M^2} - \frac{\tilde{M}_Z^2}{M^2}\right)\right) \bar{\psi} \gamma^\mu \left[\frac{\tau^3}{2} \frac{(1 - \gamma^5)}{2}\right. \\ &\quad \left. - Q s_\theta^2 \left(1 + \frac{c_\theta}{s_\theta} z_{z\gamma} \frac{\square_Z}{M^2}\right)\right] \psi Z_\mu - e \bar{\psi} \gamma^\mu Q \psi \left(1 - \frac{z_\gamma}{2} \frac{\square}{M^2}\right) A_\mu,\end{aligned}\tag{60}$$

where again we use the convention that  $\square_Z$  does only operate on  $Z$  and  $Q = \frac{\tau^3}{2} + \frac{B-L}{2}$ .

We see that the photon-fermion interaction at zero momentum correctly predicts  $e$  as the physical value of the electric charge. The Fermi constant  $G_F$  can be measured from the  $\mu$  decay, still at zero momentum. We have:

$$\begin{aligned}\frac{G_F}{\sqrt{2}} &= \frac{e^2}{8s_\theta^2} \left(1 - z_w \frac{M_W^2}{M^2}\right) \frac{1}{\tilde{M}_W^2} \left(1 + z_w \frac{M_W^2}{M^2}\right) \\ &= \frac{e^2}{8s_\theta^2 c_\theta^2 M_Z^2} \left(1 + z_z \frac{M_Z^2}{M^2}\right),\end{aligned}\tag{61}$$

where we have substituted the physical masses  $M_W$  and  $M_Z$  to  $\tilde{M}_W$  and  $\tilde{M}_Z$  since they only differ by terms of  $O(M_Z^2/\overline{M}^2)$ , which are negligible in a term which is already of the same order. From eq. (61) we can define the effective Weinberg angle (see [28]):

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8s_{\theta_0}^2 c_{\theta_0}^2 M_Z^2} \Rightarrow s_{\theta_0}^2 c_{\theta_0}^2 = s_\theta^2 c_\theta^2 \left(1 + z_z \frac{M_Z^2}{\overline{M}^2}\right), \quad (62)$$

that is

$$s_{\theta_0}^2 = s_\theta^2 \left(1 + \frac{c_\theta^2}{c_{2\theta}} z_z \frac{M_Z^2}{\overline{M}^2}\right). \quad (63)$$

Notice that the  $\epsilon$  parameters, which we obtained from the correlators (41), can be also derived as in [43] by using the rescaling of the fields given in (57) and by evaluating the  $\Delta\rho$ ,  $\Delta k$  and  $\Delta r_W$  parameters [28]. The results precisely agree with those obtained in Section IV.

Now, to get the corrections to the trilinear gauge boson couplings, it is sufficient to substitute eq. (57) in eq. (53). The general expression we get is quite long, so we will not report it. We will instead specialize to the study of a particular physical process, which will allow us to simplify eq. (53) slightly, in order to get a more readable result. The process we will consider as an example is  $e^+e^- \rightarrow W^+W^-$  scattering; in this case, the  $W$  are on-shell, so that we have

$$\partial_\mu W^{\pm\mu} = 0 \quad (64)$$

thanks to the Ward identity. Limiting the study to tree level, we can either have a virtual  $\gamma$  or a virtual  $Z$  as an intermediate state (besides the neutrino exchange which is not relevant to the study of the trilinear gauge couplings). The 4-divergence of the virtual  $\gamma$  also vanishes due to the Ward identity; while the virtual  $Z$  has an approximately vanishing transverse contribution thanks to the Dirac equation, since it is coupled to external fermions of negligible mass (compared to the center of mass energy which is of order  $M_Z$ ). So we can take:

$$\partial_\mu A^\mu = 0, \quad \partial_\mu Z^\mu \simeq 0. \quad (65)$$

As a consequence all the divergence-proportional terms in the effective Lagrangian (the last three lines in eq. (53)) can be safely dropped out. In this way, taking into account eq. (63), we get the following expression for the trilinear gauge boson couplings, relevant for the



$e^+e^- \rightarrow W^+W^-$  process:

$$\begin{aligned}
\mathcal{L}_{eff}^3 = & -ie \frac{c_{\theta_0}}{s_{\theta_0}} \left( 1 + \frac{z_z}{2c_{2\theta}} \frac{M_Z^2}{\overline{M}^2} - \frac{z_w}{2} \frac{\square_+ + M_W^2}{\overline{M}^2} - \frac{z_w}{2} \frac{\square_- + M_W^2}{\overline{M}^2} \right. \\
& \left. - \frac{z_z}{2} \frac{\square_Z + M_Z^2}{\overline{M}^2} \right) (Z_{\mu\nu} W^{-\mu} W^{+\nu} + Z_\nu (W^{-\mu\nu} W_\mu^+ - W^{+\mu\nu} W_\mu^-)) \\
& + ie \left( 1 - \frac{z_w}{2} \frac{\square_+ + M_W^2}{\overline{M}^2} - \frac{z_w}{2} \frac{\square_- + M_W^2}{\overline{M}^2} + \left( \frac{z_\gamma}{2} - z_w \right) \frac{\square_A}{\overline{M}^2} \right) \\
& (A_{\mu\nu} W^{-\mu} W^{+\nu} + A_\nu (W^{-\mu\nu} W_\mu^+ - W^{+\mu\nu} W_\mu^-)) .
\end{aligned} \tag{66}$$

We see that the tensor structure of the anomalous terms is identical to that of the SM, while the coefficients of the various operators contain derivative terms. Due to the presence of these nontrivial form factors and to the fact that the fermion-gauge boson couplings are also modified, as shown in eq. (60), the comparison of the predictions of eq. (66) to the experimental data is not direct, but requires a full calculation of the  $e^+e^- \rightarrow W^+W^-$  cross-section in the GD-BESS model. However the present experimental bounds from LEP2 on the anomalous trilinear couplings [55] have errors of the order of a few percent. Since the determination of the new physics parameters entering in eq. (66) is at the level of a few permil from LEP/Tevatron, it is clear that a higher precision will be necessary in order to achieve the same accuracy. We have nevertheless checked that, taking for example the expression of  $g_1^Z$  extracted from eq. (66), and comparing with the present experimental value for  $g_1^Z = 0.984_{-0.019}^{+0.022}$ , we get bounds on the plane  $(\overline{M}, \frac{1}{G})$  which are not relevant with respect to the ones shown in Fig. 2. We have also checked that, in order to have comparable bounds, one would need an estimation of  $g_1^Z$  at the permil level.

## VI. CONCLUSIONS

We have considered a linear moose model based on the extended gauge symmetry  $SU(2)^{2N+1}$ . The model has the central link missing and a left-right symmetry along the moose. As a consequence of the missing link the  $\epsilon_3$  parameter is zero at the leading order in  $(M_Z^2/\overline{M}^2)$ , where  $\overline{M}$  is the mass scale of the new resonances. This result can also be understood in the following way: since the model describes  $N$  pairs of new gauge boson triplets degenerate in mass, the  $\epsilon_3$  contribution of the vector resonances is canceled by the axial vector one.

We have computed the low energy effective Lagrangian by eliminating the internal moose

gauge fields and extracted the electroweak precision parameters and the trilinear anomalous couplings. Since these parameters turn out to be of order  $(g/\overline{G})^2 M_Z^2/\overline{M}^2$  and since the new effective coupling  $\overline{G}$  could well be much larger than  $g$ , the possibility of a low scale  $\overline{M}$  is left open. We expect the GD-BESS model in the present formulation to become a strongly interacting theory at energies of the order of 1.7 TeV independently of the values of the model parameters as a consequence of the perturbative unitarity violation, so it is interesting, among the future developments, to study how to unitarize it. We are currently investigating the possibility to include scalars associated to the non linear  $\sigma$ -model fields. However, in this unitarized extension, we expect corrections to the  $\epsilon_i$  parameters of the same order of magnitude as the ones evaluated here so not spoiling our overall conclusions.

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